

Quiz 1 Solution

- This is a closed book 90 minutes exam. You allowed to bring one A4 hand written cheat sheet which will be collected at end of exam. It is recommended that you read the whole exam before you start solving.
- Write your name and section number on both the question and answer sheets.
- Clearly identify your control mass / control volume.

Problem 1 (25 points)

Consider the counter-flow heat exchanger shown in Figure 1 operating at steady conditions. Heat is exchanged between two streams of fluid moving opposite to each other. The mass flowrate of the hot fluid is $\dot{m}_h = 10$ kg/s and that of the cold fluid is $\dot{m}_c = 5$ kg/s. The inlet and outlet specific enthalpies of the hot fluid are $h_{h,1} = 3000$ kJ/kg and $h_{h,2} = 2500$ kJ/kg. The inlet specific enthalpy of the cold fluid is $h_{c,1} = 1500$ kJ/kg. In this problem changes in kinetic and potential energies are neglected.

- (a) Assuming that the heat exchanger is perfectly insulated, what is the outlet specific enthalpy of the cold fluid $h_{c,2}$?
- (b) What is the rate of entropy transport into the heat exchanger across its boundary due to heat and work?
- (c) Assuming specific entropy is known at both inlets and both outlets, $s_{h,1}, s_{h,2}, s_{c,1}, s_{c,2}$, are given. What is the rate of entropy generation by irreversibility \dot{S}_{irr} ? Is \dot{S}_{irr} positive, negative, zero? Explain.

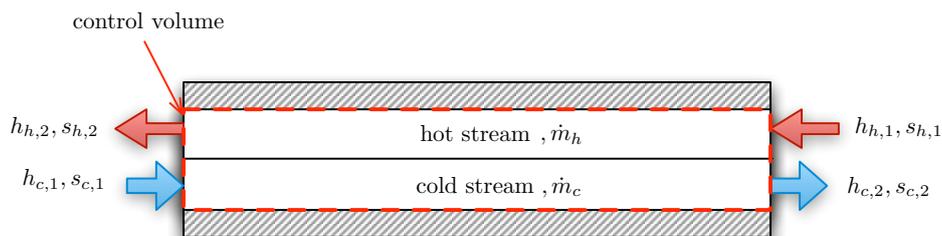


Figure 1: Schematic for problem 1.

Problem 1 Solution

(a) First law for a control volume at steady state, insulated $Q^{\leftarrow} = 0$, and in absence of all kinds of work, and neglecting changes in kinetic and potential energy

$$\begin{aligned} \dot{m}_c(h_{c,2} - h_{c,1}) + \dot{m}_h(h_{h,2} - h_{h,1}) &= 0 \Rightarrow h_{c,2} = h_{c,1} + \frac{\dot{m}_h}{\dot{m}_c}(h_{h,1} - h_{h,2}) \\ \Rightarrow h_{c,2} &= 1500 + 2(3000 - 2500) = 2500 \text{ kJ/kg} \end{aligned}$$

(b) Entropy transport due to work is zero. so $\dot{S}_W^\leftarrow = 0$ ALWAYS. Since there is not heat transfer, entropy transport is due to heat zero; $S_Q^\leftarrow = 0$.

(c) Entropy balance

$$\dot{m}_c(s_{c,2} - s_{c,1}) + \dot{m}_h(s_{h,2} - s_{h,1}) = \dot{S}_{irr}$$

It must be positive because the process is irreversible due to the heat transfer from the hot stream to the cold stream. In order to reverse this heat transfer, we must put work, which permanently changes the environment.

Problem 2 (25 points)

Consider the power generation engine shown in Figure 2. The engine operates in a thermodynamic cycle. (Thermodynamic cycle: final state is identical to initial state.) During this cycle, the engine produces work W_{cycle}^\rightarrow while receiving heat Q_H^\leftarrow for a high-temperature reservoir. The entropy transported into the engine due to heat transfer Q_H^\leftarrow is S_H^\leftarrow .

(a) Relate W_{cycle}^\rightarrow to Q_H^\leftarrow .

(b) By performing entropy balance, show that the above cyclic process is impossible.

(c) Explain that the only way the entropy balance for the cyclic engine is satisfied, the engine must reject entropy S^\rightarrow to the surrounding. How much should S^\rightarrow be? How can it be practically done ?

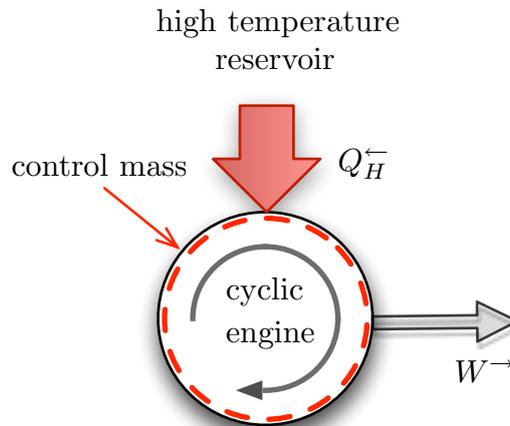


Figure 2: Schematic for problem 2.

Problem 2 Solution

(a) For an engine operating in a cycle $\Delta E = 0$ since $E_2 = E_1$, then $W^\rightarrow = Q^\leftarrow$.

(b) Entropy balance $S_2 = S_1$, so that

$$\begin{aligned} S_2 - S_1 &= S_{Q_H}^\leftarrow + S_{irr} \\ \Rightarrow 0 &= S_{Q_H}^\leftarrow + S_{irr} \end{aligned}$$

Since both terms on the right hand side of the above equation are positive, their sum cannot be zero.

(c) We need to remove entropy by rejecting heat Q_L^{\rightarrow} to the surrounding so that entropy balance becomes

$$0 = S_{Q_H}^{\leftarrow} - S_{Q_L}^{\rightarrow} + S_{irr} \Rightarrow S_{Q_L}^{\rightarrow} = S_{Q_H}^{\leftarrow} + S_{irr}$$

It is impossible for a cyclic engine to produce work while interacting with only one thermal reservoir.

Problem 3 (25 points)

We consider isothermal expansion of 10 kg of air in the the piston cylinder arrangement of Figure 3. Air is modeled as an ideal gas with $R = 0.287$ kJ/kg.K. During the process from initial (equilibrium) state 1 to final (equilibrium) state 2, heat is added to the gas by conduction across a wall of thickness $h = 0.5$ cm, area $A = 0.1$ m², and thermal conductivity $k = 50$ W/(m.K). On the gas side of the conducting wall, the gas temperature is $T = 50$ °C and on the outer side of the wall, the temperature is $T_o = 100$ °C .

(a) If heat is conducted steadily through the wall into the gas from time $t_1 = 0$ to times $t_2 = 10$ sec, what is the heat gained by gas Q^{\leftarrow} ? [Hint: conduction heat flux is $\mathbf{q} = -k\nabla T$ and assume linear temperature distribution on conducting wall.]

(b) Can we neglect changes in kinetic energy and changes in potential energy, i.e. $KE_2 - KE_1$ and $PE_2 - PE_1$? Why?

(c) What is the work done by the gas during the expansion process? If the initial volume is $V_1 = 0.01$ m³, what is the final volume V_2 , temperature T_2 , and pressure p_2 ?

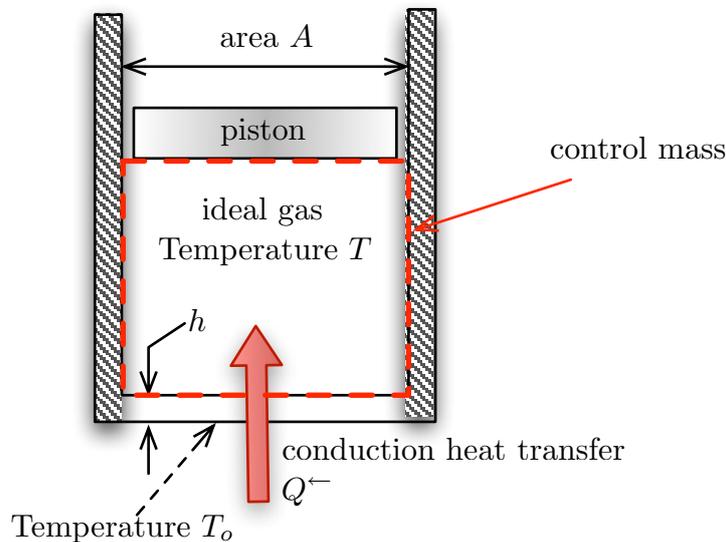


Figure 3: Schematic for problem 3.

Problem 3 Solution

(a) The heat gain per unit area per unit time is $q = k(T_o - T_i)/h = 50(100 - 50)/0.005 = 500$ kJ/m².s. The rate of heat gain is then $qA = 500 \times 0.1 = 50$ kW. The heat transferred over the period of 10 s is then $Q^{\leftarrow} = qA(t_2 - t_1) = 500$ kJ.

(b) The changes in potential and kinetic energy may be neglected if $|\Delta K.E.| \ll Q^{\leftarrow}$

and $|\Delta P.E.| \ll Q^-$. Some of you stated that they should be less than ΔU , which is zero in our problem, the process being isothermal. So this last statement is wrong. For equilibrium or quasi-equilibrium initial and final states, the gas velocities are small, and when multiplied by the gas density, which is small, yield small $|\Delta K.E.|$. Note that for (exact) equilibrium states, $K.E._1 = 0$ and $K.E._2 = 0$. As for changes in potential energy $|\Delta P.E.| = \rho g(V_2 - V_1)$ which for small ρ and $|V_2 - V_1|$ may be neglected with respect to Q^- .

(c) First law of thermodynamics for a control mass between state 1 and state 2 is

$$E_2 - E_1 = Q^- + W^-$$

Since the process is isothermal $U_2 - U_1 = 0$, neglecting changes in kinetic and potential energy, we get $W^- = Q^- = 500$ kJ. The work done by a system during an isothermal process is $W^- = mRT \ln \frac{V_2}{V_1} \Rightarrow V_2 = V_1 e^{\frac{W^-}{mRT}} = 0.01 \times 1.715 = 0.0175$ m³. Ideal gas law leads to $p_2 = 54055$ pa.

Problem 4 (35 points)

In this problem, we investigate the thermodynamics of friction within a fluid. In particular, we consider a layer of lubricant (oil) of viscosity μ and density ρ . The oil layer separates a fixed wall (which could be the cylinder wall) from a moving wall (which could be the piston wall). The problem is steady and the upper wall velocity is V , and the oil layer thickness is h . The velocity distribution inside the oil may be obtained from fluid mechanics to be $u(y) = V \frac{y}{h}$.

(a) The friction (viscous) force per unit area acting on (the lower side of) the moving wall is $\tau_w = -\mu (\partial u / \partial y)_{y=h}$. In which direction does this force pull the upper wall with respect to its motion? Explain the molecular origins of this force.

(b) What is the rate of work done on the oil assuming that the dimensions of the oil layer in the x and z are respectively 1 m and 1 m.

(c) If the oil enters the left side (i.e. at $x = 0$) at temperature T , what is its temperature at exit (i.e. at $x = 1$ m). Assume the oil layer to be thermally insulated at the upper and lower wall sides. Assume also that the specific heat of oil, c_v , does not change over the ranges of temperatures involved in the problem.

(d) Given initial and final equilibrium states, is the work done by the friction force path-dependent? Support your answer by one or more scenarios.

(e) Is this process reversible? Explain.

Problem 4 Solution

(a) Friction pulls the wall in a direction opposite to its motion. Friction is due to the tangential component of momentum transfer between adjacent layers of molecules that due to their temperature are randomly oscillating about a mean position.

(b) The rate of work done on oil per unit area is given by

$$\frac{\dot{W}}{A} = \frac{1}{A} \frac{\partial}{\partial t} \int \mathbf{F} \cdot d\mathbf{x} = \int \frac{\mathbf{F}}{A} \cdot d\mathbf{V}$$

Notice that the force per unit area on the oil is opposite to the force per unit area on the upper wall (action/reaction) so that $\mathbf{F}/A = -\tau_w \hat{\mathbf{x}}$. The fluid velocity at $y = h$ is $\mathbf{V} = V \hat{\mathbf{x}}$

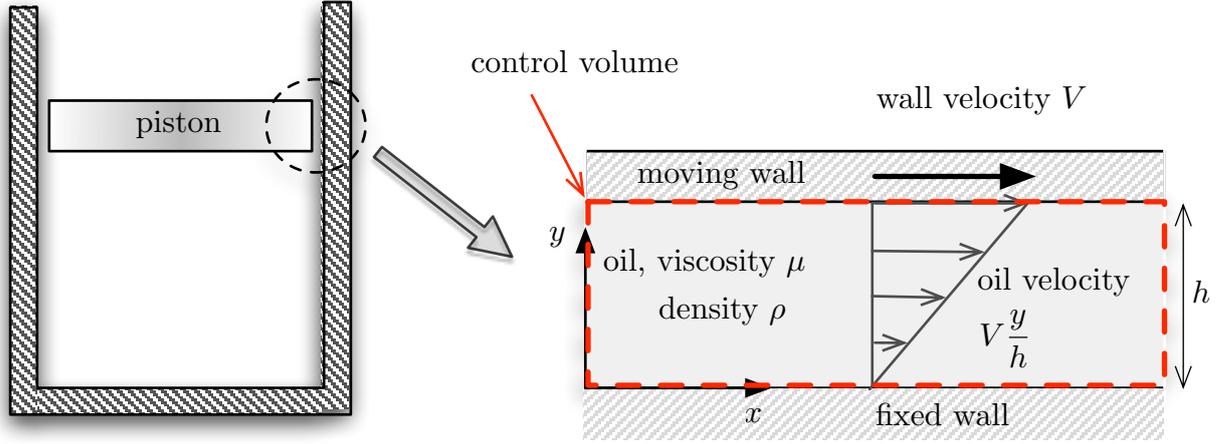


Figure 4: Schematic for problem 4.

so that

$$\frac{\dot{W}}{A} = -\tau_w V = \mu V \left(\frac{\partial u}{\partial y} \right)_{y=h} = \frac{\mu V^2}{h}$$

The rate of work done on the oil is then

$$\dot{W}^{\leftarrow} = \frac{\dot{W}}{A} A = \frac{\mu V^2 A}{h}$$

where $A = 1 \text{ m}^2$.

(c) For the control volume chosen to contain the oil of thickness h and extending from $x = 0$ to $x = 1 \text{ m}$, applying the first law for a control volume and noting that the flow is steady, and noting that changes in kinetic energy and potential energy between inlet and outlet is zero, then

$$\dot{W}^{\leftarrow} = \dot{m}(h_2 - h_1)$$

where $\dot{m} = \rho V h / 2$. Considering oil to be compressed liquid for which $h \simeq h(T)$, then $h_2 - h_1 = c_p(T_2 - T_1)$ so that

$$T_2 = T_1 + 2 \frac{\dot{W}^{\leftarrow}}{\rho V h c_p} = T_1 + 2 \frac{\mu V}{\rho c_p h^2}$$

where it was assumed that c_p does not change over the range $[T_1, T_2]$.

(d) The work done by friction on the oil is path dependent. We could replace the work effect by heat transfer into the oil that yields the same temperature rise. In the first scenario $\dot{W}^{\leftarrow} = \mu V^2 / h$, $Q^{\leftarrow} = 0$ and in the second scenario $\dot{W}^{\leftarrow} = 0$, $Q^{\leftarrow} = \mu V^2 / h$. In both cases we have the same inlet and outlet states.

(e) The process is irreversible because if we isolate the system, which amounts to stopping the upper wall, there will internal heat transfer in the oil until a final uniform temperature is reached at equilibrium.